Progression in Calculations (2020/21)

"Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject."

National Curriculum in England 2014 ***Department for Education***

This calculation policy is a guide for all staff at Ludlow Primary School and forms part of the mathematics policy.

It is designed to be used alongside any teaching resources that teachers wish to use.

All staff have access to Maths-No Problem resources which provides lessons and a host of ideas and activities to develop mastery in Mathematics. These resources are excellent ways to support the learning of mathematics and should be tailored to support the needs of the pupils. Staff are also encouraged to access the NCETM and White Rose Websites for further ideas and guidance. In EYFS, Development Matters statements are referred to; to inform planning and progress towards meeting the Early Learning Goals:

All teachers have access to the schemes of work from the White Rose Maths Hub. This module also uses the Singapore Maths Methods and is affiliated to the workings of the New Mathematics Curriculum that is running throughout the school. Where appropriate, staff are encouraged to base their planning around these recommended modules. However, it should be emphasised that all planning should take account of the requirements of the pupils in terms of where they are in their learning and how they can achieve successful outcomes. Teachers are responsible for making these judgements.

The White Rose Maths schemes of work provide sequential programmes of study that are underpinned by promoting fluency in number. They emphasise that all pupils must have a thorough grounding in the four basic rules of number before progressing on to the next level. This complete understanding gives pupils more confidence in dealing with number activities and in turn, leads to mastery of the four operations.

Whilst the calculation policy guidance document is separated into year group phases, these are intended to be used only as a guide and it is the teachers' professional judgement as to when the pupils move on to the next phase.

Progression in Calculations (2020/21)

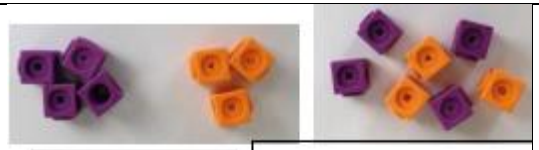
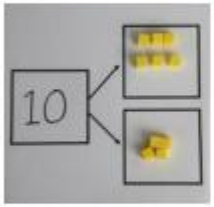
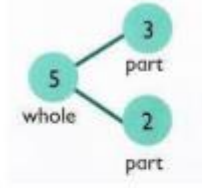
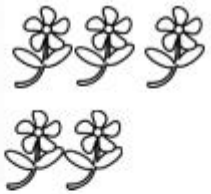
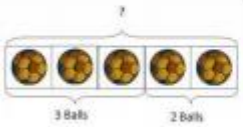
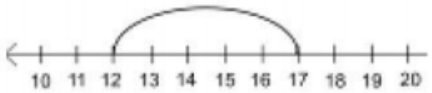
	EYFS	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Saying which number is one more than a given number. Finding the total number of items in two groups by counting all of them. Finding the total by starting at the bigger number and counting on. Introduce the part part whole model.	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Column method – no regrouping.	Column method – regrouping. (Up to 3 digits)	Column method – regrouping. (Up to 4 digits)	Column method – regrouping. (with more than 4 digits) Decimals – with the same amount of decimal places	Column method – regrouping. Decimals – with the different amounts of decimal places
Subtraction	Taking away using objects or drawing and crossing out. Saying which number is one less than a given number. Subtracting two single digit numbers by counting back. Introduce the part part whole model.	Taking away ones Counting back Find the difference Part part whole model Make 10	Counting back Finding the difference Part whole model Make 10 Column method – no regrouping	Column method – regrouping. (Up to 3 digits)	Column method – regrouping. (Up to 4 digits)	Column method – regrouping. (with more than 4 digits) Decimals – with the same amount of decimal places	Column method – regrouping. Decimals – with the different amounts of decimal places
Multiplictaion	Problem solving - doubling	Doubling Counting in multiples	Doubling Counting in multiples Repeated addition Arrays – showing commutative multiplication	Counting in multiples Repeated addition Arrays – showing commutative multiplication	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication (multi digit numbers multiplied by a 2 digit number)
Division	Problem solving – halving and sharing.	Sharing objects into groups Division as grouping	Division as grouping Division within arrays	Division within arrays Division with a remainder Short Division (2 digits by 1 digit- concrete and pictorial)	Division within arrays Division with a remainder Short Division (up to 3 digits by 1 digit- concrete and pictorial)	Short Division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2 digit number interpret remainders as whole numbers, fractions as required)

Progression in Calculations (2020/21)

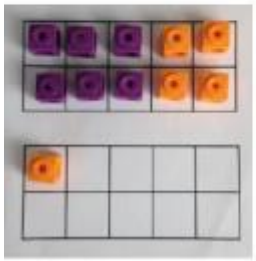
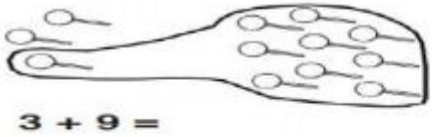
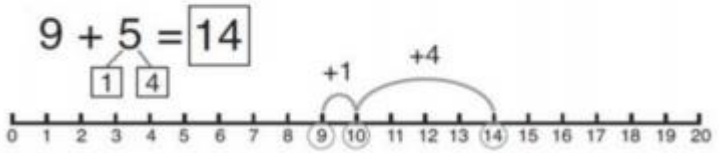
Addition

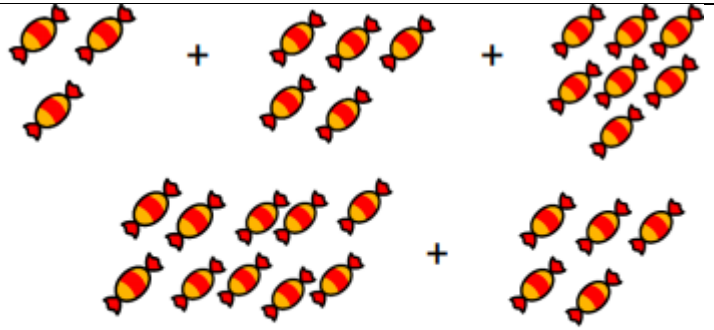
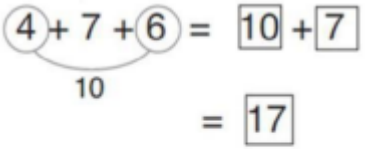
Objectives and strategies	Concrete	Pictorial	Abstract
Saying which number is more than a given number	 Use Numicon to add one more  Use cubes	 Use pictures to add one more	4 and 1 makes ☐ 4 + 1 = ☐
Finding a total number of items in two groups by counting all	 Use Numicon  Use objects	 Use pictures to add 2 groups	3 and 4 makes ☐ 3 + 4 = ☐
Finding the total number of items in two groups by counting on	 Use Numicon to count on  Use blocks	 Counting on using pictures	5 + 3 = ☐ Move into abstract (holding larger numbers in head)

Progression in Calculations (2020/21)

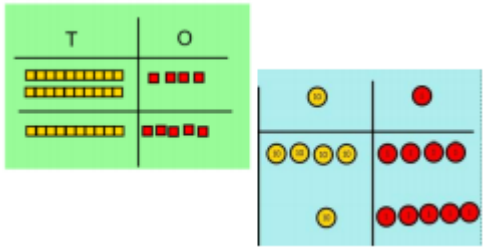
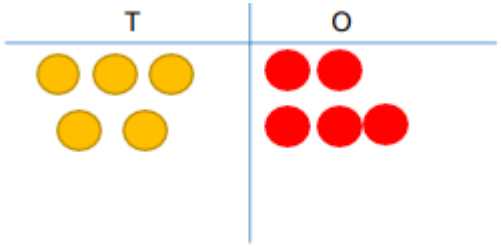
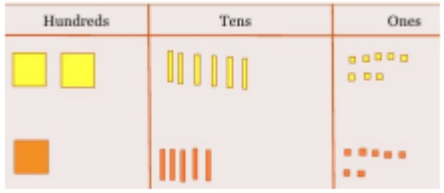
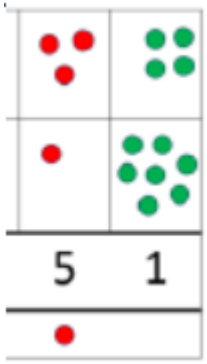
Combining two parts to make a whole: part-whole model	  Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

Progression in Calculations (2020/21)

<table>
<tr>
<td>Regrouping to make 10.</td>
<td> 
6 + 5 = 11
 
Start with the bigger number and use the smaller number to make 10.</td>
<td> 
3 + 9 =
Use pictures or a number line. Regroup or partition the smaller number to make 10.
  </td>
<td>7 + 4= 11

If I am at seven, how many more do I need to make 10. How many more do I add on now?</td>
</tr>
<tr>
<td>Adding three single digits</td>
<td>4 + 7 + 6= 17
Put 4 and 6 together to make 10. Add on 7.
 
Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.</td>
<td> 
Add together three groups of objects. Draw a picture to recombine the groups to make 10.</td>
<td> 
Combine the two numbers that make 10 and then add on the remainder.</td>
</tr>
</table>

Progression in Calculations (2020/21)

Column method without regrouping	Add together the ones first then add the tens. Use the Base 10 equipment first before moving onto place value counters. 24 + 15 = 44 + 15 = 	After practically using the base 10 equipment and place value counters, children can draw the counters using a place value frame to help them to solve additions. 32 + 23 = 	Add the ones first, then the tens, then the hundreds. 2 2 3 + 1 1 4 3 3 7
Column method with regrouping	This process is to be done with the base 10 equipment to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. Add, re-group 10 ones for a ten and 10 tens for a hundred.  Progressing to place value counters. Make both numbers on a place value grid.	Children draw a pictorial representation of the place value frame and counters to further support their learning and understanding re-grouping the ten underneath the equals line. 	Start by partitioning the numbers before moving on to formal written methods clearly show the re-grouping. 25 + 48 = 20 + 5 40 + 8 60 + 13 = 73 Add the ones first, then the tens, then the hundreds.

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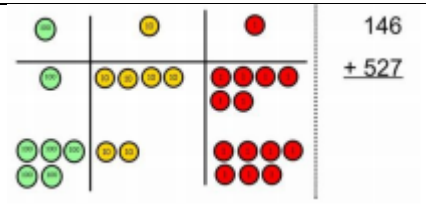
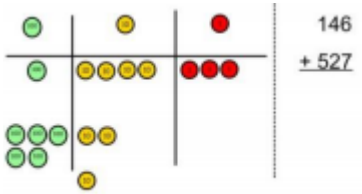
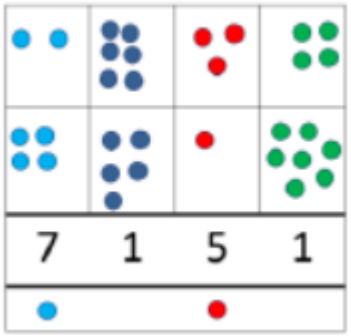
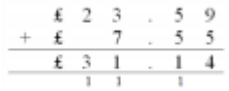
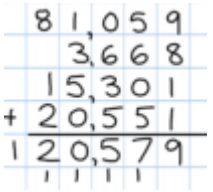
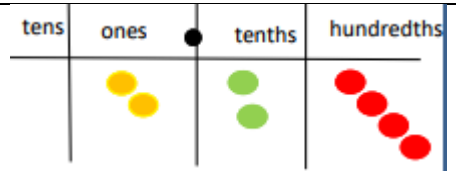
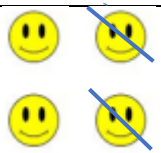
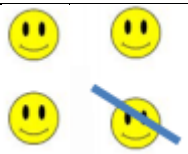
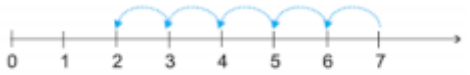
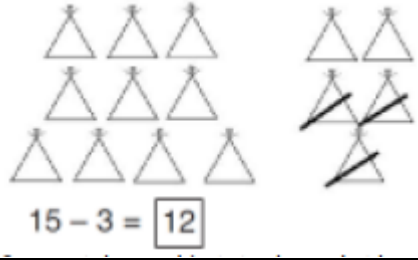
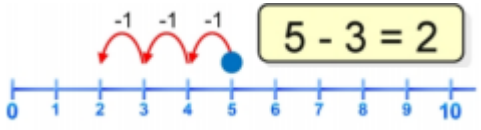
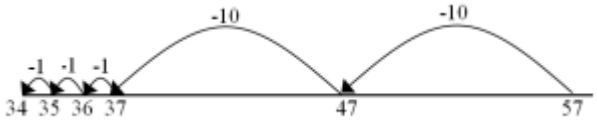
	 Add up the ones and re-group 10 ones for one 10.  Add up the rest of the columns, re-grouping the 10 counters from one column for the next place value column until every column has been added. As children move on to decimals, money and decimals place value counters can be used to support learning.		H T O 536 + 85 621 1 1 As the children move on, introduce decimals with the same number of decimal places and different places. Money can be used here. 72.8 + 54.6 127.4 1 1   Insert zeros for place holders.

			2 3 . 3 6 1 9 . 0 8 0 5 9 . 7 7 0 + 1 . 3 0 0 9 3 . 5 1 1 2 1 2

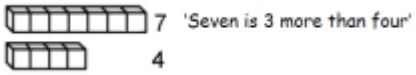
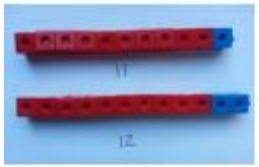
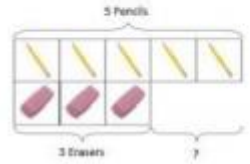
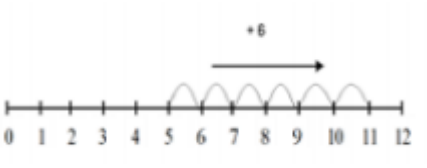
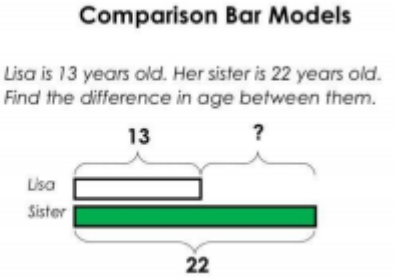
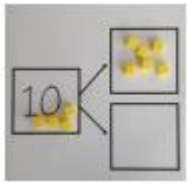
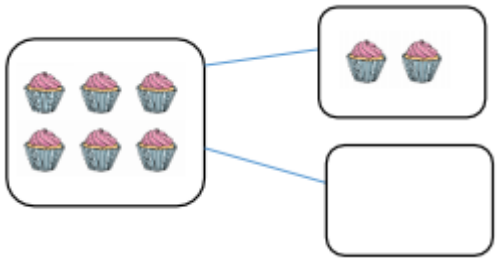
Subtraction

Objectives and strategies	Concrete	Pictorial	Abstract
Subtraction as take away	 Physically taking away  Tractor pull	 Crossing out	4 take away 2 makes ☐ 4 – 2 = ☐
Saying which number is one less than a given number	 Physically removing one item "Yum"	 Crossing out one	4 take away 1 makes ☐ 1 less than 4 is ☐ 1 fewer than 4 is ☐

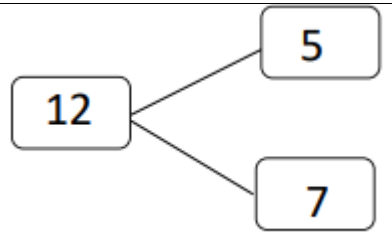
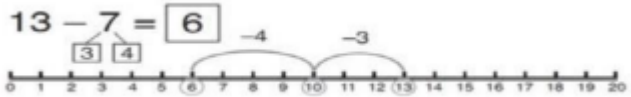
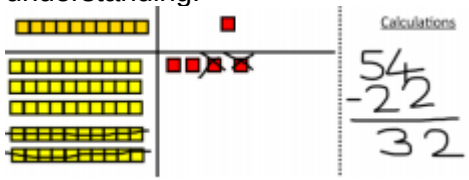
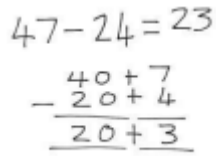
Progression in Calculations (2020/21)

Subtracting two single digit numbers by counting back	 Physical number line	 Counting back on number line	9 - 4 = ☐ Put larger number in head and count back
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away. 6 – 4 = 2 	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	7 – 4 = 3 6 = 8 – 2 18 – 3 = 15
Counting back	Move objects away from the group, counting backwards.  Make the larger number in your subtraction. Move the beads along the bead string as you count backwards in ones.	Count back in ones using a number line.  This can progress all the way to counting back using two 2 digit numbers.  	Put 13 in your head, count back 4. What number are you at?

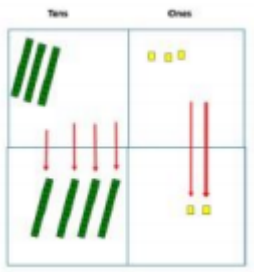
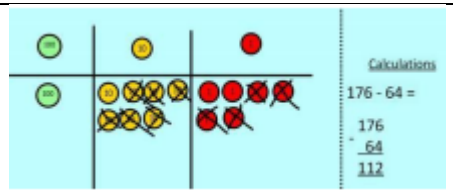
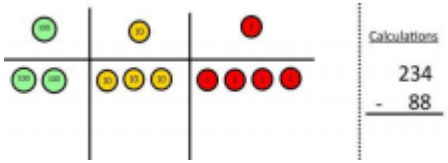
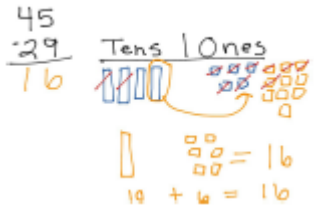
Progression in Calculations (2020/21)

			
Find the difference	Compare amounts and objects to find the difference.   Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	Count on using a number line to find the difference.  Draw bars to find the difference between 2 numbers. 	Hannah has 23 sweets, her sister has 15 sweets. Find the difference between the number of sweets. Ben has 12 marbles and his brother has 5. How many more marbles does Ben have than his brother?
Part Whole Model	Link to addition – use the part whole model to help explain the inverse between addition and subtraction.  If 10 is the whole and 6 is one of the parts. What is the other part?	Use a pictorial representation of objects to show the part whole model. 	Move to using numbers within the part whole model.

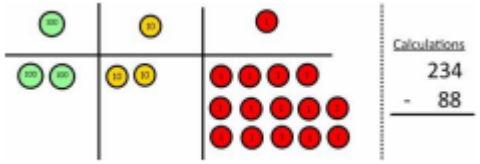
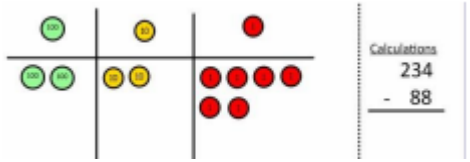
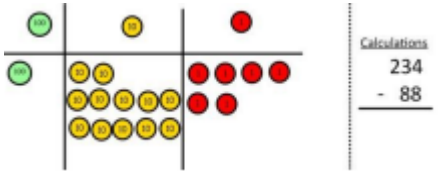
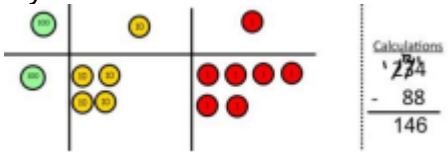
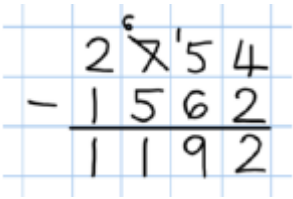
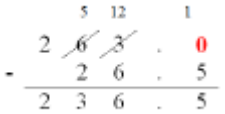
Progression in Calculations (2020/21)

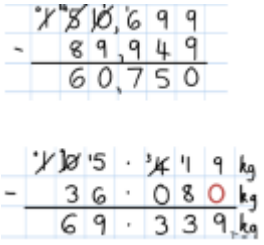
	10 – 6 =		
Make 10	14 – 5 Make 14 on the ten frame. We will partition the 5. Take away the 4 first to make 10 and then take away 1 more so you have taken away 5. 	Use a number line. 13 – 7 = Start at 13. Partition the 7 into a 3 and a 4 so can take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. 	16 - 8 = Partition the 8. How many do we take off to reach the next 10? How many do we have left to take off?
Column method without regrouping	Use the base 10 equipment to make the bigger number then take the smaller number away.	Draw the Base 10 or place value counters alongside the written calculation to support understanding. 	Intermediate step of partitioning. 

Progression in Calculations (2020/21)

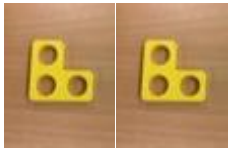
	 Show how you partition numbers to subtract. Again make the larger number first. 		This will lead to a clear written column subtraction. T O 
Column method with regrouping	Use Base 10 to start with before moving onto place value counters. Start with one regrouping before moving onto subtractions with 2 regroupings then onto 3. Make the larger number with the place value counters  Start with the ones, can I take 8 from 4? I need to regroup one of my tens for 10	Children draw the Base 10 equipment or the place value counters to 	Children can start their formal written method by partioning the number into clear plave value columns. 836 – 254 = 582 H T O 700 ~~800~~ 130 6 -200 50 4 ---------- 500 80 2 ----------

Progression in Calculations (2020/21)

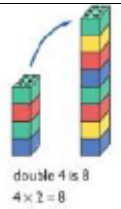
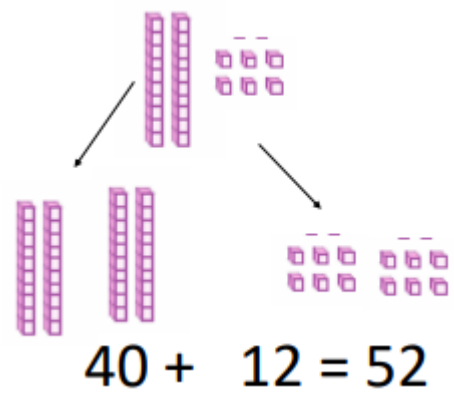
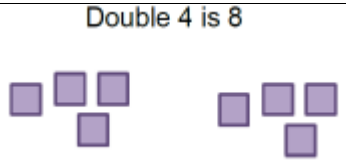
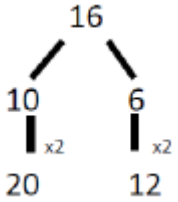
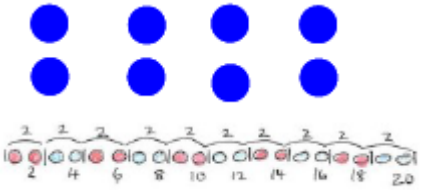
	ones.  Now I can subtract my ones.  Now look at the tens, can I take away 8 tens? I need to regroup 1 hundred for 10 tens.  Now I can take away 8 tens and complete my subtraction.  Show how the concrete method links to the written method alongside your workings. Cross out the numbers when		The children then progress to formal written methods. 728 – 582 H T O 6 7 1 2 8 - 5 8 2 --- 1 4 6 --- This will lead to subtracting any number  including decimals Use zeros for place holders 

	regrouping and show where and how we write the new amount.		

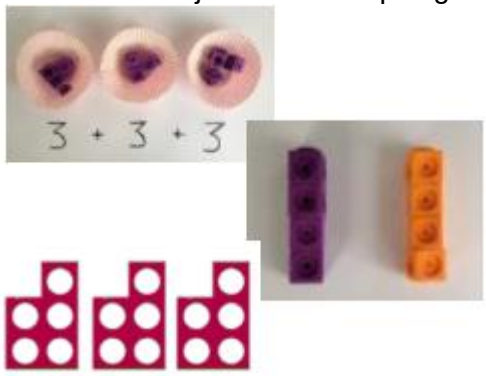
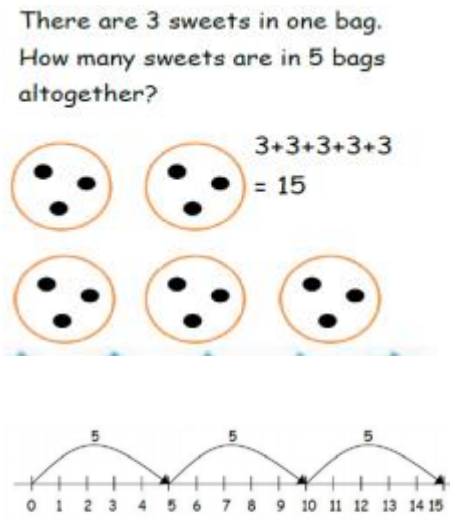
Multiplication

Objectives and strategies	Concrete	Pictorial	Abstract
Problem solving - doubling	 I have 3 pears. Can you double the number of pears? 	 Can you double the numicon shape? 	What is double 3? Double 3 is ☐
Doubling	Use practical activities to show how to double a number.	Draw pictures to show how to double a number.	Partition a number and then double each part before recombining it back together.

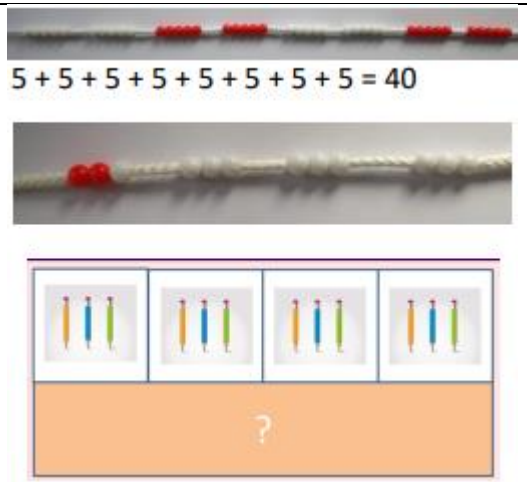
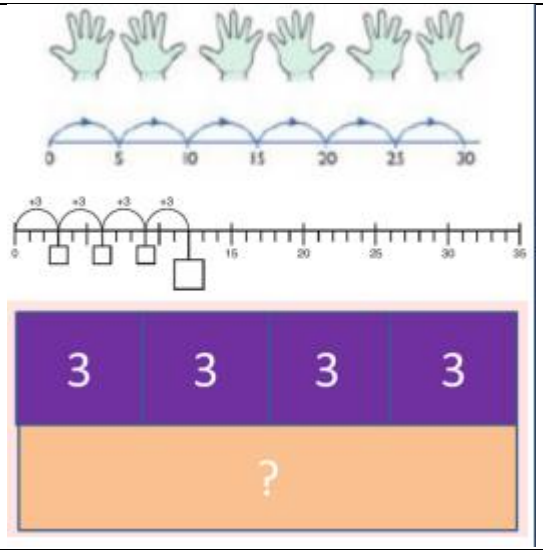
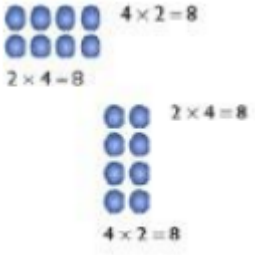
Progression in Calculations (2020/21)

	 Model doubling using the Base ten equipment: Double 26 = 		
Counting in multiples	Count in multples supportes by concrete objects in equal groups 	Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30

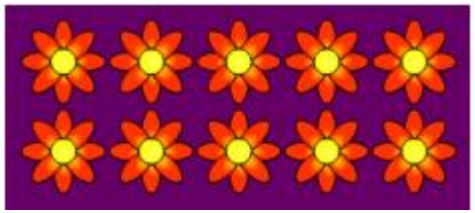
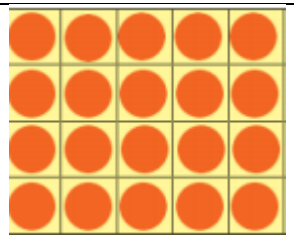
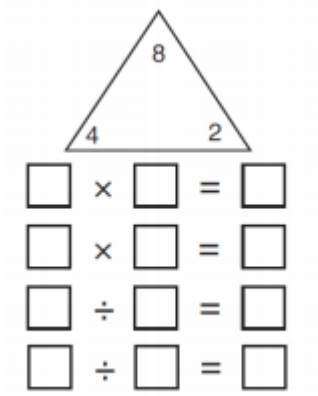
Progression in Calculations (2020/21)

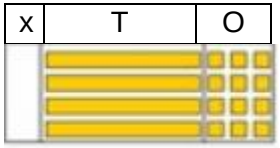
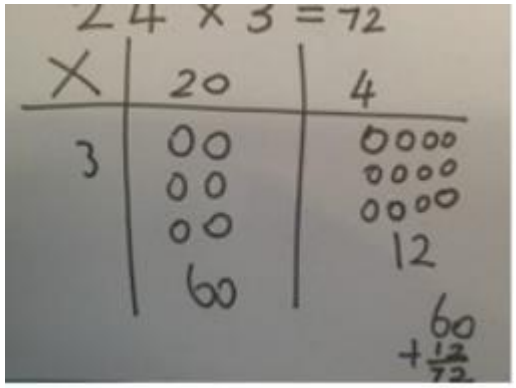
			
Repeated addition	Use different objects to add equal groups 	Use pictorial including number lines to solve problem 	Write addition asentences to describe objects and pictures. 
Counting in multiples from 0 (repeated addition)	Count the groups as children skip count. Use bar models.	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers.

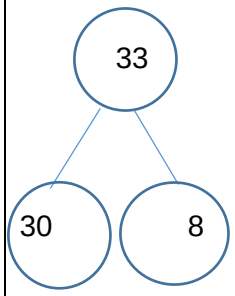
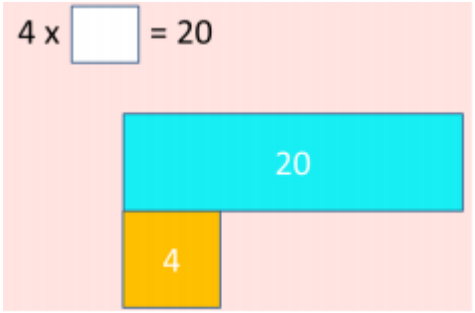
Progression in Calculations (2020/21)

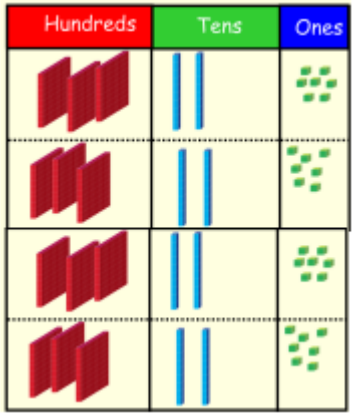
			0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$
Arrays showing commutative multiplication	Create arrays using counters/cubes to show multiplication sentences  	Draw arrays in different rotations to find commutative multiplication sentences  Link arrays to areas of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Progression in Calculations (2020/21)

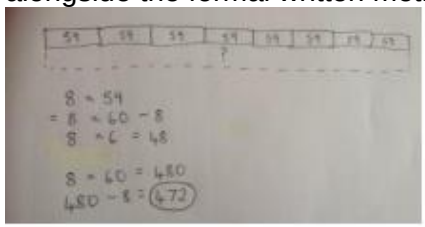
	And find answers to 2 lots of 5, 3 lots of 2 etc.  Pupiuls should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multplication does not affect the answer.  		
Using the inverse. This should be taught alongside division so pupils learn how they work			Show all 8 related fact family sentences.

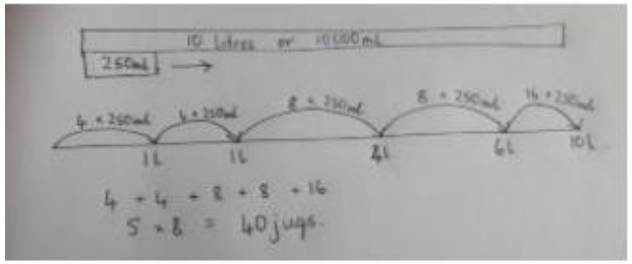
alongside each other.			2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2
Partitioning	Use base ten to move towards a more compact method. 4 x 13 = 	Children can represent their work with place value couters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking:  Draw part whole models	Children use partitioning and use the multiplication facts that they know to help them by making numbers 10 x smaller to multiply then make them 10 x bigger in the answer. 33 x 8 = 30 x 8 = 240 3 x 8 = 24 240 + 24 = 264

		 Bar models are used to explore missing numbers 	
Column multiplication	Children continue to be supported by base ten equipment. This is inititially done where ther is no regrouping ie 321 x 2 = Prgressing to re-grouping always mulyiply the ones column first. The corresponding long multiplication is modelled alongside.	Bar models and number lines can support learners when solving problems with multiplication	Start with long multiplication, reminding children about lining up their numbers clearly in columns. Initially, Children to write out what they are solving next to their answer to help them understand the process.



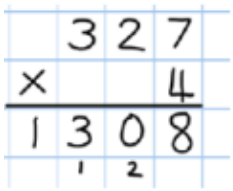
alongside the formal written methods.



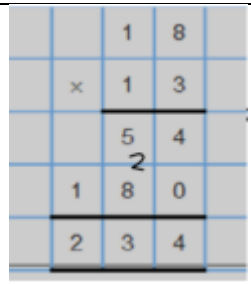
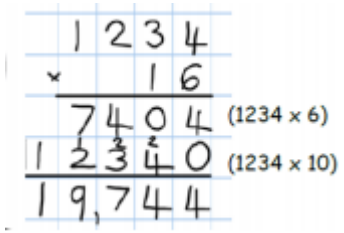


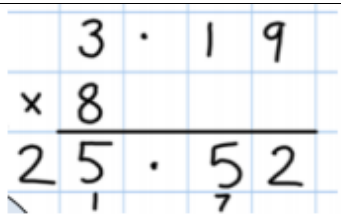
32
x 24
8 (4 x 2)
120 (4 x 30)
40 (20 x 2)
600 (20 x 30)
768

327
x 4
28
80
1200
1308

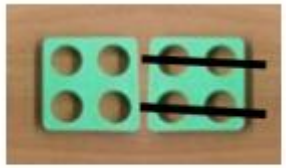


This may lead to a compact method.

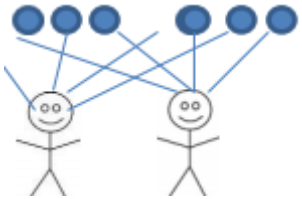
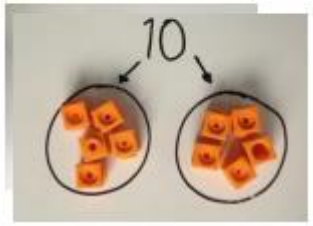
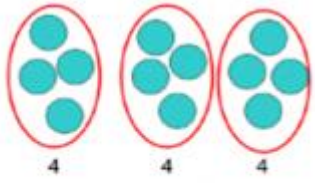
			  Multiplying decimals up to 2 decimal places by a single digit: Remind children that the single digit belongs to the ones column. Line up the decimal points in the question and the answer.

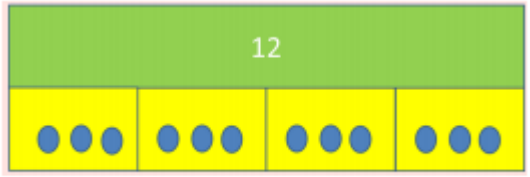
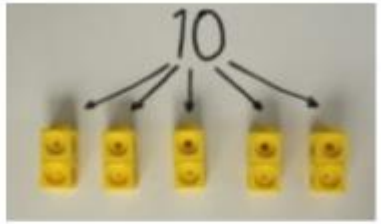
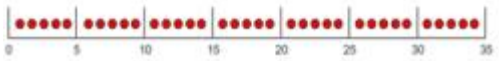
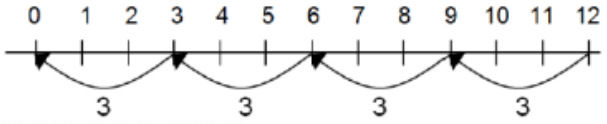
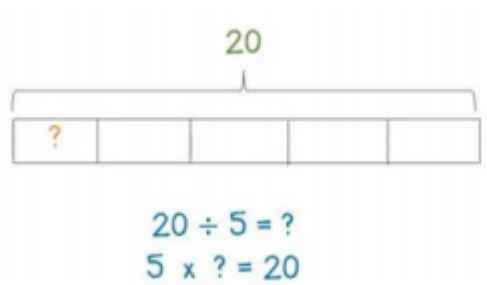
Division

Objectives and strategies	Concrete	Pictorial	Abstract
Problem solving - halving	 I have 4 pencils. I give half of these pencils to a friend. Can you cut the cake/pizza in half?	 Cross off half of the holes on the Numicon. How many holes are left?	Half of 8 is ☐ What is half of 8?

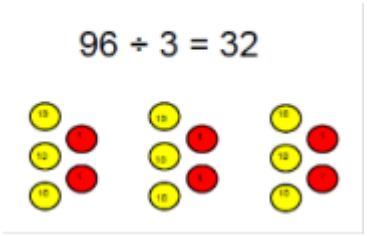
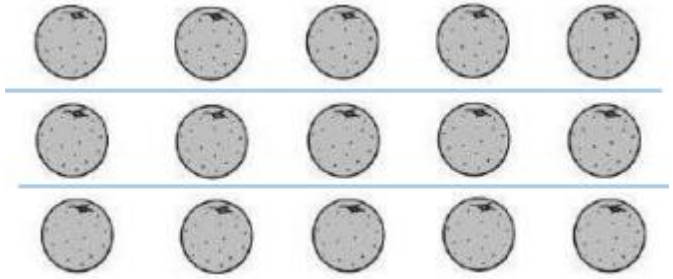
Progression in Calculations (2020/21)

Problem solving - sharing	 Share these 6 pears between 3 children in the class.	Show how these marbles can be shared between two children 	What is 8 shared between 2? Ben has eight marbles and he wants to share them equally with his friend, Sam. How many marbles to they get each?
Sharing objects into groups	I have 10 cubes. Can you share them equally into 2 groups?  	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 Sharing:  12 shared between 3 is 4	Share 9 sweets between 3 children 9 ÷ 3 = 3

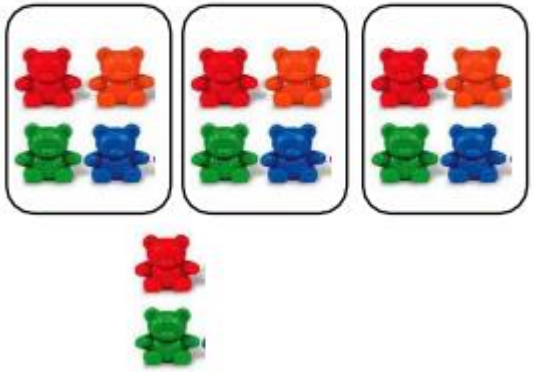
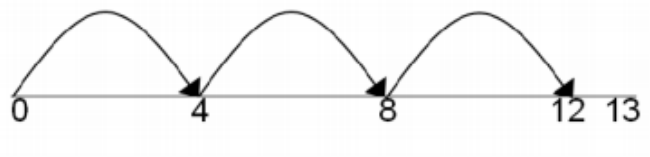
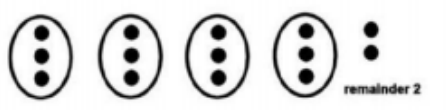
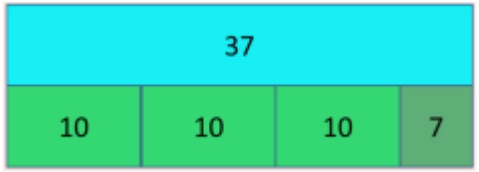
Progression in Calculations (2020/21)

		Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Progression in Calculations (2020/21)

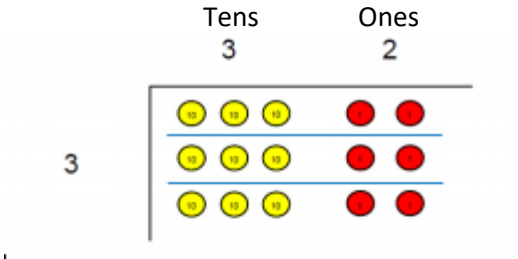
	Use the Base Ten equipment or place value counters: 24 divided into groups of 6 = 4 		
Division with arrays.	Link division to multiplication by creating an array and thinking about the number sentences that can be created.  Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences. 	Find the inverse of multiplictaion and division sentences by creating four linking family number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

Progression in Calculations (2020/21)

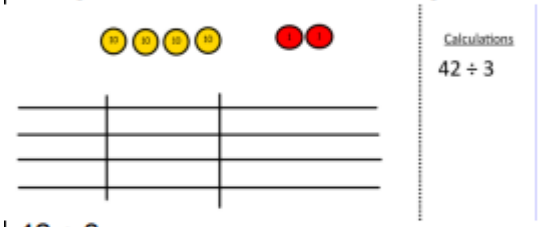
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  	Complete written divisions and show the remainder using r. 

Progression in Calculations (2020/21)

Short division

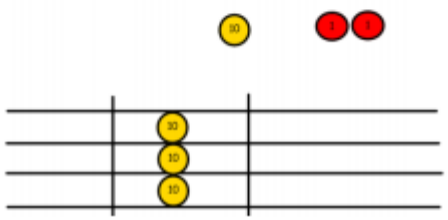


Use place value counters to divide using the bus stop method alongside

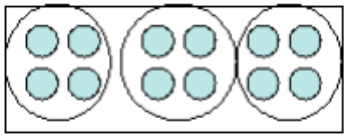


42 ÷ 3=

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 2\ 1\ 8 \\ 4\overline{)\,8\ 7\ 2} \\ \quad\ 3 \end{array}$$

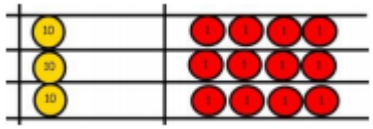
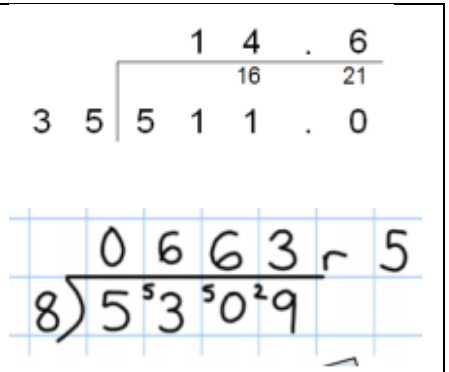
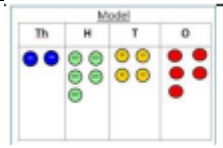
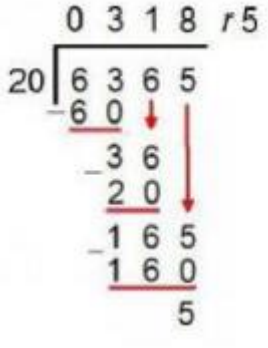
Move onto divisions with a remainder.

$$\begin{array}{r} 8\ 6\ \text{r}\ 2 \\ 5\overline{)\,4\ 3\ 2} \\ \quad\ 3 \end{array}$$

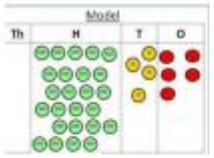
Move onto divisions with remainders expressed as fractions.

Finally move into decimal places to divide the total accurately for appropriate contexts.

Progression in Calculations (2020/21)

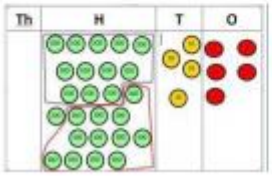
	We regroup this ten for ten ones and then share the ones equally among the groups.  We look how much is in 1 group so the answer in 14.		
Long Division	 2544 ÷ 12 How many groups of 12 thousands do we have? None Regroup 2 thousands for 20 hundreds.	Instead of using physical counters, students can draw the counters and circle the groups on a whiteboard or in their books. Use this method to explain what is happening and as soon as they have understood what move on to the abstract method as this can be a time consuming process.	

Progression in Calculations (2020/21)



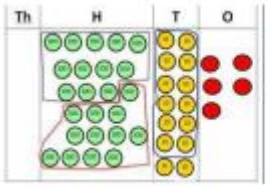
$$12\overline{)2544}\quad 0$$

How many groups of 12 are in 25 hundreds? 2 groups. Circle them.
We have grouped 24 hundreds so can take them off and we are left with one.



$$12\overline{)2544}\quad \begin{array}{r} 02 \\ 24 \\ \hline 1 \end{array}$$

Regroup the 1 hundred for 10 tens so now we have 14 tens. How many groups of 12 are there in 14? 1 remainder 2.



$$12\overline{)2544}\quad \begin{array}{r} 021 \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$

Regroup the 2 tens for 20 ones so now we have 24 ones. How may groups of 12 are in 24? 2

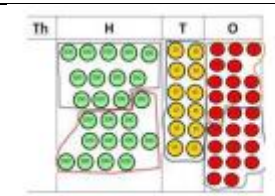
Express remainders as fractions

Express remainders as decimals

432 ÷ 15 becomes

$$\begin{array}{r} 28\cdot8 \\ 15\overline{)432\cdot0} \\ 30\downarrow \\ \hline 132 \\ 120\downarrow \\ \hline 120 \\ 120 \\ \hline 0 \end{array}$$

Answer: 28·8

	 0 2 1 2 12 | 2544 24 14 12 24 24 0		